# Pre-class Warm-up!!!

Which of the following mappings $R^2 \to R^2$ = functions
are one-to-one?
Which are onto?

| | 1 - 1 | Onto |
|---|---|---|
| a. $T(x,y) = (x+y,0)$ | No | No |
| b. $T(x,y) = (x, 2y)$ | Yes | Yes |

$f: \mathbb{R}^2 \to \mathbb{R}^2$ is onto means every $(x,y)$ in $\mathbb{R}^2$ can be written $(x,y) = f(u,v)$ for some $u,v$

As it says on the Canvas site, the exam on Tuesday covers everything through 8.1 since the last exam, namely
5.1 - 5.5, 4.1 - 4.4, 7.1, 7.2, 8.1

$f$ is 1-1 means if $f(x,y) = f(x',y')$ then $(x,y) = (x',y')$.

In case b. $(x,y) = T(x, \frac{1}{2}y)$ shows $T$ is onto

In case a. $(0,1)$ is not in the image of $T$ so $T$ is not onto. $T(\mathbb{R}^2)$

## Section 6.1 The geometry of maps R^2 -> R^2

We learn

- the meaning of one-to-one and onto
- Linear maps take straight lines to straight lines, hence parallelograms to parallelograms
- Polar coordinates in 2 dimensions
- Linear maps are 1-1 <=> their matrix has det $\neq 0$ <=> they are onto

Types of question:

- Find whether the given map is 1-1 or onto (or both)
- Find a linear map sending a given parallelogram to another parallelogram

In the book, linear maps f : R^n -> R^m are mappings given by matrix multiplication:
f(v) = Av
where v is a vector in R^n and A is an m x n matrix.

Example: f(x,y) = (x+y, x-y) is linear (see below). Only +, −, multiplication by scalars are allow.

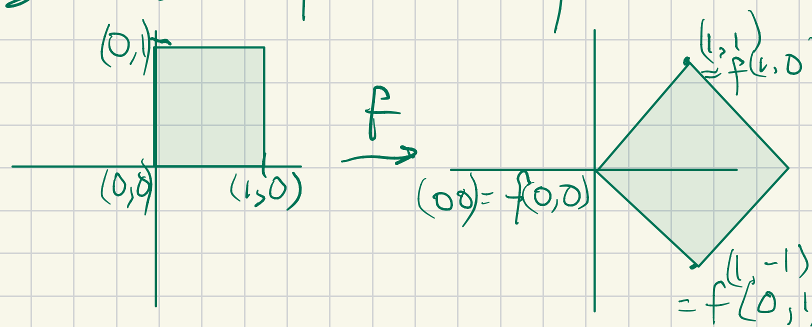
Example: Let $A = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$. Then for any vector $\begin{bmatrix} x \\ y \end{bmatrix}$ we get a linear map

$$f\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = A\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x+y \\ x-y \end{bmatrix}$$

Theorem. Linear maps take straight lines to straight lines, or single points.

Proof. Take a straight line $c(t) = u + tv$ through $u$, in direction $v$. Apply a matrix $A$:
$A\,c(t) = A(u+tv) = Au + t\,Av$
is the line passing through $Au$, in direction $Av$. $\square$

Describe the f on the left



Question:

Which matrix A expresses the mapping
$f(x,y) = (2x + 3y, 4x + y)$
in the matrix form $f(v) = Av$ ?

✓ a. $A = \begin{bmatrix} 2 & 3 \\ 4 & 1 \end{bmatrix}$

b. $A = \begin{bmatrix} 2 & 4 \\ 3 & 1 \end{bmatrix}$

c. $A = \begin{bmatrix} 1 & 3 \\ 4 & 2 \end{bmatrix}$

d. $A = \begin{bmatrix} 3 & 1 \\ 2 & 4 \end{bmatrix}$

e. None of the above

2. What about $f(x,y) = (x^2 + y^3, x^4 + y)$ ?

not linear.

How to find $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

Compute $f(1,0) = (2,4)$
$f(0,1) = (3,1)$

This means $\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} a \\ c \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$

$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} b \\ d \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$

Examples:

1. Determine if the mapping T : R^2 -> R^2 is 1-1 and/or onto.

T(x,y) = (x+y,0); T(x,y) = (2x+y, y).

2. Find a linear map T that sends the parallelogram with vertices
(0,0), (1,1), (1,2), (2,3)
to the parallelogram with vertices
(0,0), (-1,0), (-1,2), (-2,2)

We want $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ so that

$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$ $\quad$ $\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$

so $a+b=-1$ $\quad$ $a+2b=-1$

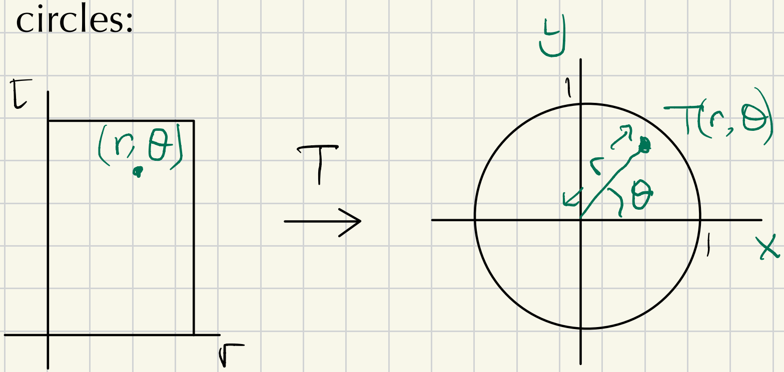
$c+d=0$ $\quad$ $c+2d=2$

Solve these.

Planar polar coordinates (r, t) refer to points with (x,y)-coordinates (r cos t, r sin t) and can be regarded as a transformation R^2 -> R^2 given by
T(r,t) = (r cos t, r sin t)

T is not 1 - 1.

It can be regarded as mapping rectangles to circles:




$$T(r, \theta) = (r\cos\theta, r\sin\theta)$$